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Third Semester B.E. Degree Examination, June/July 2023
Advanced Mathematics – I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Find the modulus and amplitude of $\frac{1}{1 - \cos \alpha + i \sin \alpha}$. (07 Marks)
- b. Express $\frac{(3+i)(1-3i)}{2+i}$ in the form of $x + iy$. (06 Marks)
- c. Find the cube roots of $1 - i$. (07 Marks)
- 2 a. Find the n^{th} derivative of $\frac{x}{(x+1)(2x+1)}$. (07 Marks)
- b. Find the n^{th} derivative of $\cos x \cos 3x \cos 5x$. (06 Marks)
- c. If $y = a \cos(\log x) + b \sin(\log x)$ show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (07 Marks)
- 3 a. Show that the following pairs of curves intersect orthogonally $r_1 = a(1 + \sin \theta)$ and $r_2 = a(1 - \sin \theta)$. (07 Marks)
- b. With usual notations prove that $\tan \phi = r \frac{d\theta}{dr}$. (06 Marks)
- c. Expand $\sqrt{1 + \sin 2x}$ by Maclaurin's series upto the term containing x^4 . (07 Marks)
- 4 a. State and prove Euler's theorem on homogeneous functions. (07 Marks)
- b. If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)
- c. If $u = x + 3y^2 - z^3$, $v = 4x^2 yz$, $w = 2z^2 - xy$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$. (07 Marks)
- 5 a. Obtain a reduction formula for $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n being a positive integer. (07 Marks)
- b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dy \, dx$. (06 Marks)
- c. Evaluate $\int_{-1}^1 \int_{x-z}^{x+z} \int_x^z (x + y + z) \, dy \, dx \, dz$. (07 Marks)

- 6 a. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (07 Marks)
- b. Express $I = \int_0^1 x^{3/2} (1-x)^{1/2} dx$ in terms of Gamma – Beta function. (06 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$. (07 Marks)
- 7 a. Solve $(2x + y + 1) dx + (x + 2y + 1) dy = 0$. (07 Marks)
- b. Solve $\frac{dy}{dx} + y \cot x = \cos x$. (06 Marks)
- c. Solve $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. (07 Marks)
- 8 a. Solve $(D^3 - 6D^2 + 11D - 6) y = 0$. (06 Marks)
- b. Solve $y'' + y = e^{-x} + \sin x$. (07 Marks)
- c. Solve $y'' - 4y = x^2$. (07 Marks)
